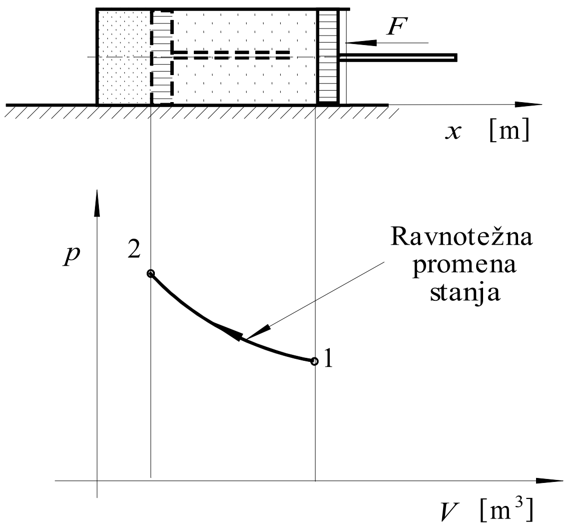
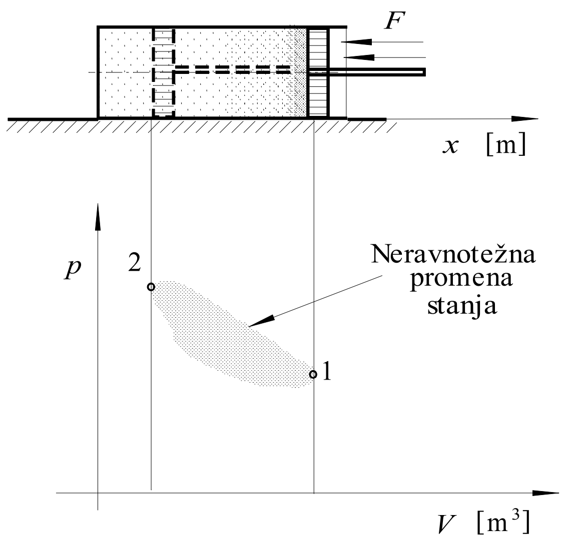
8. PROMENE STANJA IDEALNOG GASA U ZATVORENOM I MAKROSKOPSKI NEPOKRETNOM TERMO-MEHANIČKOM SISTEMU

Ravnotežne



- Politropne
- Nepolitropne

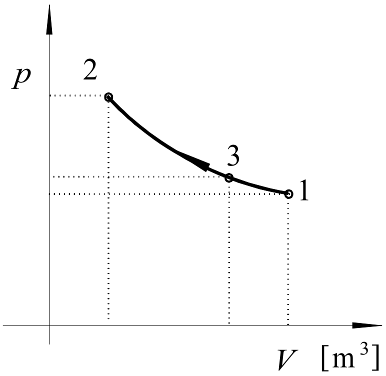
Neravnotežne



- Politropne
- Nepolitropne

8.1 Ravnotežne politropne promene stanja idealnog gasa u zatvorenom termo-mehaničkom sistemu

„Indikatorski dijagram“



Osnovna osobenost politropskih promena stanja

$$\boxed{p\,v^n = \text{idem}} \qquad \boxed{n = \text{idem}}$$

n - eksponent politropske promene stanja idealnog gasa

$$p_1\,v_1^n = p_2\,v_2^n = p_3\,v_3^n$$

Pomoću jednačine stanja idealnog gasa

$$p\,v = R\,T \;\rightarrow\; v = \frac{R\,T}{p} \;\text{ i }\; p = \frac{R\,T}{v}$$

1. $$p\,v^n = p\left(\frac{R\,T}{p}\right)^n = \text{idem} \qquad p^{1-n}T^n = \text{idem} \quad \Rightarrow \quad \boxed{pT^{\frac{n}{1-n}} = \text{idem}}$$

2. $$p\,v^n = \frac{R\,T}{v}\,v^n = \text{idem} \quad \Rightarrow \quad \boxed{T\,v^{n-1} = \text{idem}}$$

- Kako odrediti predatu količinu toplote, odnosno izvršen zapreminski rad?

Predata količina toplote

1) $p\,v^n = \text{idem}$

2) $p\,v = R\,T$

3) $\delta q + \delta w_V = \mathrm{d}u \;\rightarrow\; \delta q - p\mathrm{d}v = c_V\,\mathrm{d}T$

1) $\rightarrow$ $\mathrm{d}(p\,v^n) = 0$

$v^n\,\mathrm{d}p + n\,p\,v^{n-1}\mathrm{d}\,v = 0 \;\; / :v^{n-1}$

$v\,\mathrm{d}p + n\,p\,\mathrm{d}\,v = 0$ ....(1’)

2) $\rightarrow$ $\mathrm{d}(p\,v) = \mathrm{d}(R\,T)$

$p\,\mathrm{d}v + v\,\mathrm{d}\,p = R\,\mathrm{d}T$ ....(2’)

(1’)- (2’) $\rightarrow$

$$p\,(n-1)\,\mathrm{d}v = -R\,\mathrm{d}T \quad \rightarrow \quad p\,\mathrm{d}v = -\frac{R}{n-1}\mathrm{d}T$$

3) $$\delta q = c_V \mathrm{d}T + p\mathrm{d}v = (c_V - \frac{R}{n-1})\,\mathrm{d}T$$

$$\delta q = \frac{c_V\,n - c_V - c_p + c_V}{n-1}\mathrm{d}T$$

$$\boxed{\delta q = c_V\,\frac{n-\kappa}{n-1}\mathrm{d}T} \quad \Rightarrow \quad \boxed{q_{1-2} = c_V\,\frac{n-\kappa}{n-1}(T_2 - T_1)}$$

Kako je

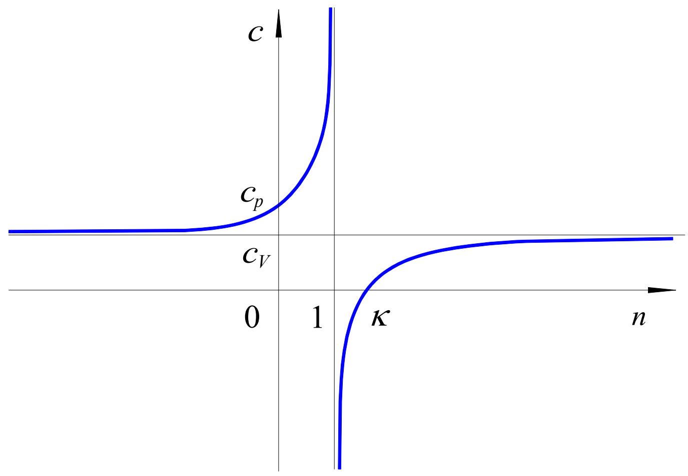
$$\delta q = c\,\mathrm{d}T \;,\; \delta q = c_V\,\frac{n-\kappa}{n-1}\mathrm{d}T \quad \mathrm{i}\;\; c = \mathrm{idem} \quad \Rightarrow$$

$\Rightarrow$ $\quad c = c_V\,\dfrac{n-\kappa}{n-1}$ - specifični topotni kapacitet politropske promene stanja idealnog gasa

- Eksponent politropske promene stanja izražen preko specifični topotnog kapacitet c

$$c = c_V\,\frac{n-\kappa}{n-1} \quad \Rightarrow \quad (n-1)\,c = c_V\,(n-\kappa) \quad \Rightarrow \quad n = \frac{c - c_p}{c - c_V}$$

- Promena specifičnog topotnog kapaciteta c politropske promene stanja idealnog gasa u zavisnosti od promene eksponenta n



Izvršeni radovi

Izvršen zapreminski rad

- Po definiciji

$$w_{V,1-2} = -\int_1^2 p\,\mathrm{d}v$$

- Korišćenjem osnovne relacije za politropske promene stanja i izražavanjem pritiska u funkciji specifične zapremine, te uvrštanjem prethodnog izraza koji definiše zapreminski rad :

$$p\,v^n = \text{idem} \;\Rightarrow\; p_1\,v_1^n = p_2\,v_2^n = p\,v^n \;\Rightarrow\; p = \frac{p_1\,v_1^n}{v^n}$$

$$w_{V,1-2} = -p_1\,v_1^n \int_1^2 v^{-n}\,\mathrm{d}v$$

- Integracijom dobijenog izraza:

$$w_{V,1-2} = -\frac{p_1\,v_1^n}{1-n}\left[v_2^{1-n} - v_1^{1-n}\right] = -\frac{p_1\,v_1}{1-n}\left[\left(\frac{v_2}{v_1}\right)^{1-n} - 1\right]$$

i korišćenjem termičke jednačine stanja za idealne gasove:

$$p\,v = R\,T \qquad p_1\,v_1 = R\,T_1$$

$$w_{V,1-2} = -\frac{RT}{1-n}\left[\left(\frac{v_2}{v_1}\right)^{1-n} - 1\right]$$

kao i jedne od relacije za politropske promene stanja

$$T\,v^{n-1} = \text{idem} \;\Rightarrow\; T_1\,v_1^{n-1} = T_2\,v_2^{n-1} \;\Rightarrow\; \left(\frac{v_2}{v_1}\right)^{n-1} = \frac{T_1}{T_2} \Rightarrow \left(\frac{v_2}{v_1}\right)^{1-n} = \frac{T_2}{T_1}$$

sledi

$$w_{V,1-2} = \frac{R\,T_1}{n-1}\left(\frac{T_2}{T_1} - 1\right)$$

$$\boxed{w_{V,1-2} = \frac{R}{n-1}\left(T_2 - T_1\right)}$$

- Na drugi način

$$\delta q + \delta w_V = \mathrm{d}u$$

$$c\,\mathrm{d}T + \delta w_V = c_V\,\mathrm{d}T \quad\Rightarrow\quad \delta w_v = c_V \mathrm{d}T - c_V\,\frac{n-\kappa}{n-1}\mathrm{d}T$$

$$\delta w_V = \frac{n\,c_V - c_V - n\,c_V + c_p}{n-1}\mathrm{d}T \quad\Rightarrow\quad \boxed{w_{V,1-2} = \frac{R}{n-1}(T_2 - T_1)}$$

Izvršen tehnički rad

- Po definiciji

$$w_{\text{teh,1-2}} = \int_1^2 v \,\mathrm{d}p$$

- Korišćenjem osnovne relacije za politropske promene stanja i izražavanjem pritiska u funkciji specifične zapremine, te uvrštanjem prethodnog izraza koji definiše zapreminski rad:

$$p_1 v_1^n = p v^n \;\Rightarrow\; v = \frac{v_1 p_1^{\frac{1}{n}}}{p^{\frac{1}{n}}}$$

$$w_{\text{teh,1-2}} = v_1 p_1^{\frac{1}{n}} \int_1^2 p^{\frac{1}{n}} \mathrm{d}p = \frac{v_1 p_1^{\frac{1}{n}}}{1-\frac{1}{n}} \left[p_2^{1-\frac{1}{n}} - p_1^{1-\frac{1}{n}} \right]$$

$$w_{\text{teh,1-2}} = \frac{p_1 v_1}{\frac{n-1}{n}} \left[\left(\frac{p_2}{p_1} \right)^{1-\frac{1}{n}} - 1 \right] = n \cdot \frac{p_1 v_1}{n-1} \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

i korišćenjem termičke jednačine stanja za idealne gasove i kao i jedne od relacije za politropske promene stanja

$$p_1 v_1 = R T_1$$

$$p_1 T_1^{\frac{n}{1-n}} = p_2 T_2^{\frac{n}{1-n}} \qquad \Rightarrow \qquad \left(\frac{p_2}{p_1}\right) = \left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}}$$

$$w_{\text{teh,1-2}} = n \frac{R T_1}{n-1} \left[\frac{T_2}{T_1} - 1 \right]$$

$$\boxed{w_{\text{teh,1-2}} = n \frac{R}{n-1} \left[T_2 - T_1 \right]}$$

ili

$$\boxed{w_{\text{teh,1-2}} = n\, w_{v,1-2}}$$

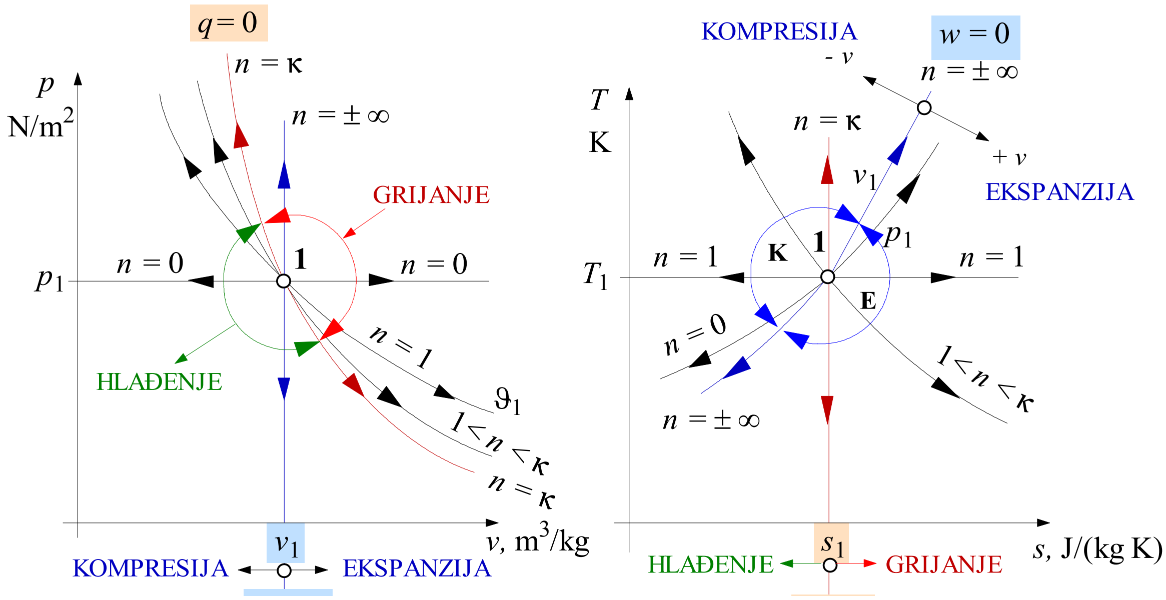
- Na drugi način

$$\delta q + \delta w_{\text{teh}} = \mathrm{d}h$$

$$\delta w_{\text{teh}} = c_p \,\mathrm{d}T - c_V \frac{n-\kappa}{n-1} = \frac{c_p n - c_p - n\, c_V + c_p}{n-1} \mathrm{d}T$$

$$\delta w_{\text{teh}} = n \frac{R}{n-1} \mathrm{d}T$$

$$\boxed{w_{\text{teh,1-2}} = n \frac{R}{n-1} \mathrm{d}T}$$



Prikaz politropnih promena stanja idealnog gasa u $p-v$ i $T-s$ koordinatnim sistemima

- **Izobarska politropska promena**, $p = \text{idem}$

$$\underline{n=0} \quad p\,v^0 = \text{idem} \quad c = c_p$$

$$\delta q = T\mathrm{d}s \qquad \delta q = c\mathrm{d}T$$

$$\delta q + \delta w_{\text{teh}} = \mathrm{d}h \quad \rightarrow \quad c\,\mathrm{d}T = c_p\,\mathrm{d}T \quad c = c_p$$

$$c_p \mathrm{d}T = T\,\mathrm{d}s$$

$$\mathrm{d}s = c_p \frac{\mathrm{d}T}{T} \quad \rightarrow \quad s_2 - s_1 = c_p \ln\frac{T_2}{T_1}$$

$$T_2 = T_1 \exp\left[\frac{s_2 - s_1}{c_p}\right] \quad \rightarrow \quad T = T_1 \exp\left[\frac{s - s_1}{c_p}\right]$$

- **Izotermaska politropska promena**, $T = \text{idem}$, $c = \pm\infty$

$$n = 1 \quad p\,v = R\,T \qquad p = \frac{p_1\,v_1}{v} = \frac{\text{idem}}{v}$$

- **Adijabatska politropska promena** $\delta q = 0$, $n = \kappa$,

a ako je i ravnotežna $\Rightarrow$ Izentropska politropska promena stanja $\mathrm{d}s = 0$, $s = \text{idem}$

$$\delta q + \delta w_{\text{teh}} = \mathrm{d}h = c_p\,\mathrm{d}T \quad \rightarrow \quad w_{\text{teh},1\text{-}2} = c_p\,(T_2 - T_1)$$

$$\delta q + \delta w_V = \mathrm{d}u = c_v\,\mathrm{d}T \quad \rightarrow \quad w_{V,1\text{-}2} = c_V\,(T_2 - T_1)$$

Za sve politropne promene važi

$$w_{\text{teh},1\text{-}2} = n\, w_{V,1\text{-}2}$$

U ovom slučaju

$$w_{\text{teh},1\text{-}2} = \kappa\, w_{V,1\text{-}2}$$

pa je

$$\frac{w_{\text{teh},1-2}}{w_{V,1-2}} = \kappa = \frac{c_p}{c_V} = \gamma$$

- **Izohorska politropna promena stanja**, $v = \text{idem}$

$$p\, v^n = \text{idem} \;\Rightarrow\; p^{\frac{1}{n}}\, v = \text{idem} \;\Rightarrow\; v = \frac{\text{idem}}{p^{\frac{1}{n}}} \;\text{ i }\; v = \text{idem} \Rightarrow\; n = \pm\infty, \;\; c = c_v$$

$$\delta q + \delta w_V = \mathrm{d}u \quad \rightarrow \quad c\,\mathrm{d}T = c_V\,\mathrm{d}T \quad \rightarrow \quad c = c_V$$

$$c_V = T\,\mathrm{d}s$$

$$\mathrm{d}s = c_p\,\frac{\mathrm{d}T}{T} \qquad \rightarrow \qquad s_2 - s_1 = c_V\,\ln\frac{T_2}{T_1}$$

$$T_2 = T_1 \exp\left[\frac{s_2 - s_1}{c_V}\right] \qquad \rightarrow \qquad T = T_1 \exp\left[\frac{s - s_1}{c_V}\right]$$